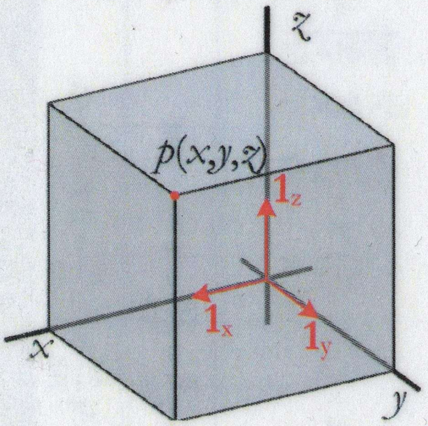
Laboratorium Teorii Pola Elektromagnetycznego

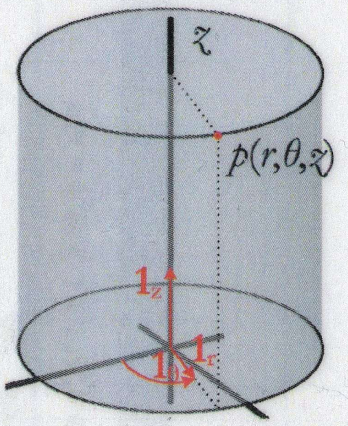
Podstawowe Zależności Wektorowe

Układy Współrzędnych

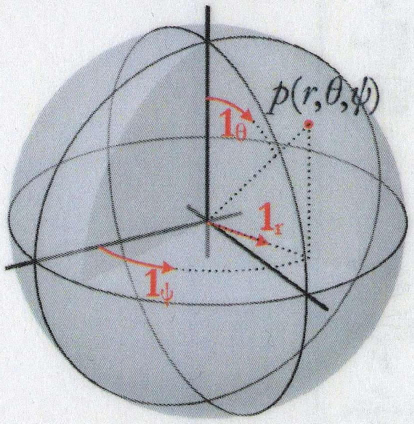
Prostokątny



Cylindryczny



Sferyczny



x, y, z ¬ współrzędne prostokątne

$x = r\cos\Theta$
$y = r\sin\Theta$
$z = z$

$x = r\sin\Theta\cos\Psi$
$y = r\sin\Theta\sin\Psi$
$z = r\cos\Theta$

Zależność między składowymi wektorów

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_z \end{bmatrix}$$

$$\begin{bmatrix} A_r \\ A_\theta \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\Psi & \cos\theta\cos\Psi & -\sin\Psi \\ \sin\theta\sin\Psi & \cos\theta\sin\Psi & \cos\Psi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_z \end{bmatrix}$$

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\Psi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\Psi & \sin\theta\sin\Psi & \cos\theta \\ \cos\theta\cos\Psi & \cos\theta\sin\Psi & -\sin\theta \\ -\sin\Psi & \cos\Psi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

Wzory algebry wektorowej

Iloczyn skalarny:

$$w = \mathbf{A}\cdot\mathbf{B} = AB\cos\alpha = A_xB_x + A_yB_y + A_zB_z$$

Iloczyn wektorowy:

$$\mathbf{F} = \mathbf{A}\times\mathbf{B} = \begin{vmatrix} 1_x & 1_y & 1_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \mathbf{1}_x(A_yB_z - A_zB_y) + \mathbf{1}_y(A_zB_x - A_xB_z) + \mathbf{1}_z(A_xB_y - A_yB_x)$$

Tożsamości wektorowe

$$\mathrm{grad}(UV) = V\,\mathrm{grad}\,U + U\,\mathrm{grad}\,V$$

$$\mathrm{div}(U\mathbf{A}) = A\cdot\mathrm{grad}\,U - U\,\mathrm{div}\,\mathbf{A}$$

$$(\mathbf{A}\cdot\mathrm{grad})\cdot\mathbf{B} = \mathbf{A}\,\mathrm{div}\,\mathbf{B} - \mathrm{rot}\,\mathbf{A}\times\mathbf{B}$$

$$\mathrm{div}\,\mathrm{grad}\,U = \nabla^2 U$$

$$\mathrm{div}(\mathbf{A}\times\mathbf{B}) = \mathbf{B}\times\mathrm{rot}\,\mathbf{A} - \mathbf{A}\,\mathrm{rot}\,\mathbf{B}$$

$$\mathrm{div}\,\mathrm{rot}\,\mathbf{A} = 0$$

$$\mathrm{rot}\,U\mathbf{A} = \mathrm{grad}\,U\times\mathbf{A} + U\,\mathrm{rot}\,\mathbf{A}$$

$$\mathrm{rot}\,\mathrm{rot}\,\mathbf{A} = \mathrm{grad}\,\mathrm{div}\,\mathbf{A} - \nabla^2\mathbf{A}$$

$$\mathrm{rot}\,\mathrm{grad}\,U = 0$$

Tożsamości wektorowe

	Kartezjański (x, y, z)	Cylindryczny (r, θ, z)
gradient	$\nabla U = \mathrm{grad}\, U = \frac{\partial U}{\partial x}\mathbf{1}_x + \frac{\partial U}{\partial y}\mathbf{1}_y + \frac{\partial U}{\partial z}\mathbf{1}_z$	$\nabla U = \frac{\partial U}{\partial r}\mathbf{1}_r + \frac{1}{r}\frac{\partial U}{\partial \theta}\mathbf{1}_\theta + \frac{\partial U}{\partial z}\mathbf{1}_z$
dywergencja	$\mathrm{div}\,\mathbf{A} = \nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$	$\nabla \cdot \mathbf{A} = \frac{1}{r}\frac{\partial (rA_r)}{\partial r} + \frac{\partial A_\theta}{\partial \theta} + \frac{\partial A_z}{\partial z}$
rotacja	$\mathrm{rot}\,\mathbf{A} = \nabla \times \mathbf{A} = \begin{vmatrix} \mathbf{1}_x & \mathbf{1}_y & \mathbf{1}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \mathbf{1}_x\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \mathbf{1}_y\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \mathbf{1}_z\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$	$\nabla \times \mathbf{A} \begin{vmatrix} \frac{1_r}{r} & 1_\theta & \frac{1_z}{r} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ A_r & rA_\theta & A_z \end{vmatrix} = 1_r\left(\frac{1}{r}\frac{\partial A_z}{\partial \theta} - \frac{\partial A_\theta}{\partial z}\right) + 1_\theta\left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right) + 1_z\frac{1}{r}\left(\frac{\partial (rA_\theta)}{\partial r} - \frac{\partial A_r}{\partial \theta}\right)$
laplasjan	$\mathrm{div}\,\mathrm{grad}\, U = \nabla^2 U = \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2}$	$\nabla^2 U = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial U}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 U}{\partial \theta^2} + \frac{\partial^2 U}{\partial z^2}$
	$\nabla^2 \mathbf{A} = \mathbf{1}_x \nabla^2 A_x + \mathbf{1}_y \nabla^2 A_y + \mathbf{1}_z \nabla^2 A_z$	$\nabla^2 \mathbf{A} = \left(\nabla^2 A_r - \frac{A_r}{r^2} - \frac{2}{r^2}\frac{\partial A_\theta}{\partial \theta}\right)\mathbf{1}_r + \left(\nabla^2 A_\theta - \frac{A_\theta}{r^2} + \frac{2}{r^2}\frac{\partial A_r}{\partial \theta}\right)\mathbf{1}_\theta + \nabla^2 A_z \mathbf{1}_z$

	Sferyczny (r, θ, ψ)
gradient	$\nabla U = \frac{\partial U}{\partial r}1_r + \frac{1}{r}\frac{\partial U}{\partial \theta}1_\theta + \frac{1}{r\sin\theta}\frac{\partial U}{\partial \psi}1_\psi$
dywergencja	$\nabla \cdot \mathbf{A} = \frac{1}{r^2}\frac{\partial (r^2 A_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial (A_\theta \sin\theta)}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial A_\psi}{\partial \psi}$
rotacja	$\nabla \times \mathbf{A} = \begin{vmatrix} \frac{1_r}{r^2\sin\theta} & \frac{1_\theta}{r\sin\theta} & \frac{1_\psi}{r} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \psi} \\ A_r & rA_\theta & rA_\psi \sin\theta \end{vmatrix}$
laplasjan	$\frac{1}{r^2}\left[\frac{\partial}{\partial r}\left(r^2\frac{\partial U}{\partial r}\right) + \frac{1}{\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial U}{\partial \theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2 U}{\partial \Psi^2}\right]$

$$\nabla^2 \mathbf{A} = \left\{\nabla^2 A_r - \frac{2}{r^2}\left[A_r + \frac{1}{\sin\theta}\left(\frac{\partial}{\partial \theta}(A_\theta \sin\theta) + \frac{\partial A_\psi}{\partial \psi}\right)\right]\right\}\mathbf{1}_r + \left\{\nabla^2 A_\theta + \frac{2}{r^2}\left[\frac{\partial A_r}{\partial \theta} - \frac{A_\theta}{2\sin^2\theta} - \frac{\cos\theta}{\sin^2\theta}\frac{\partial A_\psi}{\partial \psi}\right]\right\}\mathbf{1}_\theta + + \left\{\nabla^2 A_\psi + \frac{1}{r^2\sin\theta}\left[\frac{\partial A_r}{\partial \psi} + \mathrm{ctg}\theta\frac{\partial A_\theta}{\partial \psi} - \frac{A_\psi}{2\sin\theta}\right]\right\}\mathbf{1}_\psi$$